Code No: RT21012 **R13** SET - 1

II B. Tech I Semester Regular Examinations, Dec - 2015
PROBABILITY AND STATISTICS
(Civil Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **THREE** Questions from **Part-B**
4. Statistical tables are required

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PART –A

1. a) The density function of a random variable X is $f(x)=\begin{cases} e^{-x}, x\geq 0 \\ 0, \text{ otherwise}\end{cases}$.Find $E(X)$, $Var(X)$ (4M)

b) The first four moments of a distribution about 4 are 1,4,10 and 25 respectively. Show that the mean is 5, variance is 3, $\mu_3=0$ and $\mu_4=26$. (4M)

c) When a sample is taken from an infinite population, what happen to the standard error of the mean if the sample size is decreased from 800 to 200 . (4M)

d) Explain the types of errors in sampling. (3M)

e) In a bivariate population $\sigma_x=\sigma_y=\sigma$ and the angle between the regression lines is $\tan^{-1}(6)$, obtain the value of correlation coefficient. (4M)

f) Explain the term Statistical Quality Control. Discuss its aspects and advantages. (3M)

PART –B

2. a) If *X* is a continuous random variable with probability density function (PDF) (8M)

$$f(x)=\begin{cases} x, 0\leq x<1 \\ \frac{3}{2}(x-1)^2, 1\leq x\leq 2 \\ 0, otherwise\end{cases}$$

. Find the cumulative distribution function $F(x)$ of *X* and use it to find $P\left(\frac{3}{2}<X<\frac{5}{2}\right)$

b) Four coins are tossed 160 times. The number of times *x* heads occur is given below. (8M)

x	0	1	2	3	4
No. of times	8	34	69	43	6

Fit a binomial distribution to this data on the hypothesis that coins are unbiased.

3. a) Find the moment generating function of the Poisson distribution and, hence find the mean and variance. (8M)

b) If *X* represent the outcome when a fair die is tossed, find the MGF of *X* and hence find $E(X), Var(X)$. (8M)
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4. a) A population consists of five numbers 3,6,9,15,27. Consider all possible samples of size three that can be drawn without replacement from this population. Find (i) The population mean. (ii) The population standard deviation. (iii) The mean of the sampling distribution of the means. (iv) The standard deviation of the sampling distribution of the means. (8M)

   b) A normal population has mean of 0.1 and standard deviation of 2.1. Find the probability that mean of sample of size 900 will be negative. (8M)

5. a) The mean life of a sample of 10 electric bulbs was found to be 1456 hours with standard deviation of 432 hours. A second sample of 17 bulbs chosen from a different batch showed a mean life of 1280 hours with standard deviation of 398 hours. Is there a significant difference the means of two hatches? (8M)

   b) The following table gives the yield on 15 samples under three varieties of seeds. (8M)

A	20	21	23	16	20
B	18	20	17	25	15
C	25	28	22	28	32

Test at 5% level of significance whether the average yields of land different varieties of seeds show significance differences.

6. a) Calculate the correlation coefficient and the lines of regression from the following data: (8M)

X	22	26	29	30	31	31	34	35
Y	20	20	21	29	27	24	27	31

   b) By the method of least squares fit a parabola of the form $y = ax^2 + bx + c$ for the following (8M)

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

7. a) A darling machine bores holes with a mean diameter of 0.5230 cm and a standard deviation of 0.0032 cm. Calculate the 2-sigma and 3-sigma upper and lower control limits for mean of samples 4, and prepare a control chart. (8M)

   b) Write a short notes on : (8M)
   i) Mean chart ii) Range Chart iii) p-chart iv) C – chart

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PART –A

1. a) If X is a continuous random variable and k is a constant, then prove that (i) $Var(X+k) = Var(X)$ (ii) $Var(\mathrm{kX}) = k^2 Var(X)$. (4M)

b) Find the first four moments for the set of numbers 2,4,6,8. (4M)

c) Assuming that $\sigma = 20$, how large a random sample be taken to assert with probability 0.95 that the sample mean will not differ from the true mean by more than 3.0 points? (3M)

d) Write about (i) Null hypothesis (ii) Critical region (iii) Level of significance. (3M)

e) The tangent of the angle between the two lines of regression is 0.5 and $\sigma_x = \frac{1}{4}\sigma_y$, find the value of correlation coefficient. (4M)

f) What is a control chart? How it is designed? What purpose does it serve? (4M)

PART –B

2. a) Given probability distribution function a continuous random variable X as follows $f(x) = \begin{cases} 6x(x-1), 0 < x < 1 \\ 0, \quad otherwise \end{cases}$. Find cumulative distribution function (CDF) $F(\mathrm{x})$. (8M)

b) After correcting 50 pages, the proof reader finds that there are on the average of 2 errors per 5 pages. How many pages could one expect with 0 error, 1 error and at least 3 errors in 1000 pages of the first print of the book? (8M)

3. a) Find the moment generating function of a Binomial distribution and hence, find the mean and variance. (8M)

b) Find the first four moments about the mean from the following data . (8M)

X	1	2	3	4	5
f(x)	2	3	5	4	1

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4. a) A population consists of five numbers 4,8,12,16,20,24. Consider all possible samples of size 2 that can be drawn with replacement from this population. Find (i) The population mean. (ii) The population standard deviation. (iii) The mean of the sampling distribution of the means. (iv) The standard deviation of the sampling distribution of the means. (10M)

   b) Prove that for a random sample of size n, $X_1, X_2, \ldots, X_n$ taken from an infinite population $s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ is not unbiased estimator of the parameter σ^2 but $\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ is unbiased (6M)

5. a) In a random sample of 1000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveal a significant difference between town A and town B, so far as the proportion of wheat consumers is concerned? (8M)

   b) Two independent samples of 8 and 7 items respectively had the following values. (8M)

Sample -I	11	11	13	11	15	9	12	14
Sample -II	9	11	10	13	9	8	10	-

Is the difference between the means of samples significant?

6. a) Using the method of least square find the constants a and b such that y = a e^{bx} for the following data (8M)

X	0.0	0.5	1.0	1.5	2.0	2.5
Y	0.10	0.45	2.15	9.15	40.35	180.75

   b) Calculate the correlation coefficient for the following data: (8M)

X	56	42	72	36	63	47	55	49	38	42	68	60
Y	147	125	160	118	149	128	150	145	115	140	152	155

7. a) What is a control chart as used in Statistical Quality Control? Explain, in this connection, the term ‘Three sigma control limits’. (6M)

   b) A machine is set to deliver packets of a given weight. 10 sample size 5 each were recorded. Below are given relevant data: (10M)

Sample no	1	2	3	4	5	6	7	8	9	10
Mean($\overline{x}$)	15	17	15	18	17	14	18	15	17	16
Range(R)	7	7	4	9	8	7	12	4	11	5

Draw the control charts and comment on the state of control.

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PART –A

1. a) Prove that Poisson distribution is limiting case of Binomial distribution. (4M)

b) If X and Y are continuous random variable , then prove that (i) $E(\mathrm{X+Y}) = \mathrm{E(X)} + \mathrm{E(Y)}$ (ii) $Var(aX + b) = a^2 Var(\mathrm{X})$ where a and b are constants (4M)

c) Give the difference between the interval estimation and Bayesian estimation. (3M)

d) In a random sample of 106 workers exposed to a certain amount of radiation, 24 experienced some ill effects. Construct 99% confidence interval for the corresponding true percentage. (4M)

e) Let θ be the angle between the two regression lines *X* on *Y* and *Y* on *X.* Prove that $\tan\theta = \left(\frac{1-r^2}{r}\right)\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$, where σ_x, σ_y are standard deviations of *X* and *Y* series (3M)

f) Explain in brief how control limits are determined for (i) *P*-chart (ii) *C*-chart. (4M)

PART –B

2. a) Two dice are thrown .Let X assign to each point (a, b) in S the maximum of its numbers *i.e.,X(a,b)=max(a, b)*. Find the probability distribution, X is a random variable with *X(s)*={1,2,3,4,5,6}. Also find the mean and variance of the distribution. (8M)

b) At certain examination, 10% of the students who appeared for the paper in Statistics got less than 30 marks and 97% of the students got less than 62 marks. Assuming the distribution to be normal, find the mean and the standard deviation of the distribution. (8M)

3. a) Show that the MGF of a random variable X having the PDF $f(x) = \begin{cases} \frac{1}{3} & -1 < x < 2 \\ 0 & otherwise \end{cases}$ is given by $M_X(t) = \begin{cases} \frac{e^{2t} - e^{-t}}{3t} & t \neq 0 \\ 1 & t = 0 \end{cases}$. (8M)

b) A random variable X has the probability density function $f(x) = kx^2(1-x^3)$ where $0 \le \mathrm{x} \le 1$. Find (i) k (ii) the first 3 moments about origin (iii) the first two central moments. (8M)
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4. a) A random sample of size 64 is taken from a normal population with mean 51.4 and standard deviation 68. What is the probability that the mean of the sample will (i) exceed 52.9 (ii) fall between 50.5 and 52.3 (iii) be less than 50.6. (8M)

   b) (i)Prove that for a random sample of size n, $X_1, X_2 ....., X_n$ taken from an infinite population $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ is an unbiased estimator of the parameter σ^2. (8M)

   (ii)Determine a 95% confidence interval for the mean of a normal distribution with variance 0.25, using a sample of *n=100* values with mean 212.3.

5. a) Explain the procedure generally followed in testing of hypothesis. (6M)

   b) 4 coins were tossed 160 times and the following results were obtained (10M)

No. of Heads	0	1	2	3	4
Observed frequencies:	17	52	54	31	6

   Under the assumption that coins are balanced, find the expected frequencies of 0,1,2,3,or 4 heads, and test the goodness of fit.($\alpha = 0.05$).

6. a) Fit a curve of the form $y = ab^x$ in least square method for the following data : (8M)

X	2	3	4	5	6
Y	144	172.8	207.4	248.8	298.5

   b) Calculate the correlation coefficient and the lines of regression from the following data: (8M)

x	22	26	29	30	31	31	34	35
y	20	20	21	29	27	24	27	31

7. a) Write the advantages of Statistical Quality Control. (6M)

   b) The following data gives the number of defectives in 20 samples, containing 2000 items. (10M)

425	430	216	341	225	322	280	306	337	305
356	402	216	264	126	409	193	280	326	389

   Calculate the values for central line and control limits for *p-chart* (fraction defective Chart) .

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PART –A

1. a) If a random variable X has the probability function $f(x)=\begin{cases}\frac{1}{2}(x+1), if\ -1<x<1\\ 0\ otherwise\end{cases}$. Find the mean and Variance. (4M)

b) Prove that the mathematical expectation of the sum of n random variables is equal to the sum of their expectations, provided all the expectations exist. (4M)

c) Explain briefly Point and Interval estimations. (3M)

d) Explain briefly the χ^2 (Chi-square) test. (3M)

e) From a sample of 200 pairs of observation the following quantities were calculated. $\sum X=11.34, \sum Y=20.78$, $\sum X^2=12.16, \sum Y^2=84.96, \sum XY=22.13$ .From the above show how to compute the coefficient of the equation $Y=a+bX$. (4M)

f) Explain clearly the construction and functions of (i) $\overline{X}-chart$, (ii) $P-chart$ (iii) $C-chart$ and their control limits. (4M)

PART –B

2. a) The probability density function of a random variable X is given by (8M)

$$f(x)=\begin{cases}\frac{1}{16}(3+x)^2\ if\ -3\le x<-1\\ \frac{1}{16}(6-2x^2),\ if\ -1\le x<1\\ \frac{1}{16}(3-x)^2,\ if 1\le x\le 3\\ 0,\ elsewhere\end{cases}$$

Show that the area under the curve above x-axis is unity. Also find the mean of the distribution.

b) If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3kgs, now many students have masses (i) greater than 72kgs (ii) less than are equal to 64kgs (iii) Between 65 and 71 kg inclusive (8M)
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3. a) A random variable X has the probability density function is given by $f(x)=\begin{cases} ke^{-2x}, x \geq 0 \\ 0, \text{x} < 0 \end{cases}$ (i) find k (ii) The Moment generating function.(iii) The first four moments about the origin (8M)

b) The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 14.409 and 454.98. Calculate the moments about the mean. Also evaluate Skewness and Kurtosis. (8M)

4. a) A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs.487 with a standard deviation Rs.48. With what degree of confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between 472 to 502? (8M)

b) Find 95% confidence limits for the mean of a normality distributed population from which the following was taken 15, 17, 10, 18, 16, 16, 9, 7, 11, 13, and 14. (8M)

5. a) Time taken by the workers in performing a job by method I and method II is given below: (8M)

Method-I	20	16	26	27	23	22	-
Method-II	27	33	42	35	32	34	38

Do the data show that the variances of time distribution from population from which these samples are drawn do not differ significantly?

b) The three samples below have been obtained from normal population with equal variances. Test the at 5% level that the population means are equal. (8M)

Sample-I	8	10	7	14	11
Sample –II	7	5	10	9	9
Sample-III	12	9	13	12	14

(The table value of $F_{0.05}(v_1 = 2, v_2) = 3.88$)

6. a) Fit a straight line to the form $y = a + bx$ for the following data (6M)

x	0	5	10	15	20	25
y	12	15	17	22	24	30

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b) Calculate the correlation coefficient and the lines of regression from the following data: (10M)

X	62	64	65	69	70	71	72	74
Y	126	125	139	145	165	152	180	208

7. a) Write a short note on the utility of control charts in Statistical Quality Control. (8M)

b) The average number of defectives in 22 sampled lots 2000 rubber belts each was found to be 16%. Indicate how to construct the relevant control chart. (8M)